Attendance Problems.

1. What is the name of the point where the angle bisectors of a triangle intersect?

Find the midpoint of the segment with the given endpoints.

2. (–1, 6) & (3, 0) **3.** (–7, 2) & (–3, –8)

4. Write an equation of the line containing the points **(**3, 1**)** and **(**2, 10**)** in point-slope form.

- I can apply properties of medians of a triangle.
- I can apply properties of altitudes of a triangle.

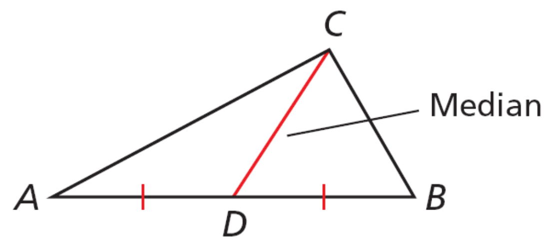
Vocabulary	
median of a triangle	centroid of a triangle
altitude of a triangle	orthocenter of a triangle

Common Core

CC.9-12.G.CO.10 Prove theorems about triangles.
CC.9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods.
CC.9-12.G.MG.3 Apply geometric methods to solve design problems.

Sketchpad: Medians in a triangle.

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.



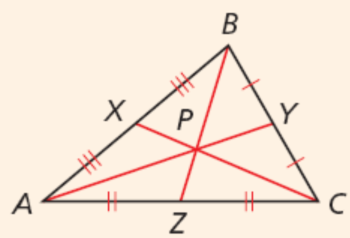
Every triangle has three medians, and the medians are concurrent.

The point of concurrency of the medians of a triangle is the **centroid of the triangle** . The centroid is always inside the triangle. The centroid is also called the *center of gravity* because it is the point where a triangular region will balance.

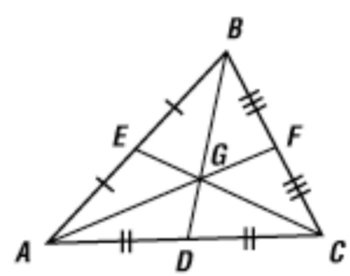
Theorem 5-3-1 **Centroid Theorem**

The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

$AP = \frac{2}{3}AY$ $\quad$ $BP = \frac{2}{3}BZ$ $\quad$ $CP = \frac{2}{3}CX$

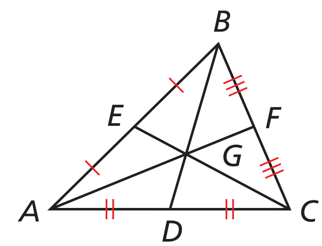


Refer to video example 1.
In ΔABC, $AF = 12$, and $GE = 4.8$. Find AG & CE.



1 Using the Centroid to Find Segment Lengths

In $\triangle ABC$, $AF = 9$, and $GE = 2.4$. Find each length.



A AG

$AG = \frac{2}{3}AF$ — Centroid Thm.

$AG = \frac{2}{3}(9)$ — Substitute 9 for AF.

$AG = 6$ — Simplify.

B CE

$CG = \frac{2}{3}CE$ — Centroid Thm.

$CG + GE = CE$ — Seg. Add. Post.

$\frac{2}{3}CE + GE = CE$ — Substitute $\frac{2}{3}$CE for CG.

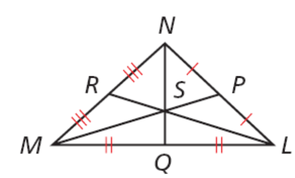
$GE = \frac{1}{3}CE$ — Subtract $\frac{2}{3}$CE from both sides.

$2.4 = \frac{1}{3}CE$ — Substitute 2.4 for GE.

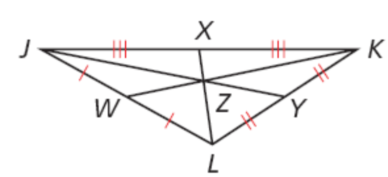
$7.2 = CE$ — Multiply both sides by 3.

Example 1. In ΔLMN, $RL = 21$ and $SQ = 4$.

A. Find LS. **B.** Find NQ.

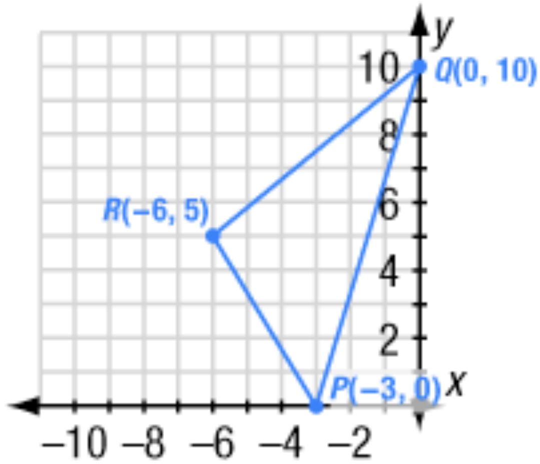


5. Guided Practice. In ΔJKL, $ZW = 7$, and $LX = 8.1$. Find KW & LZ.



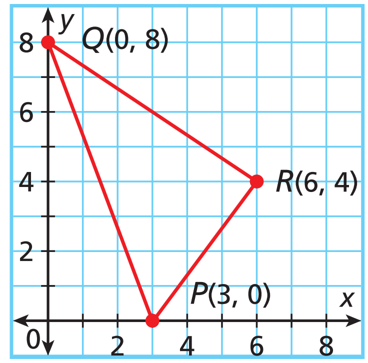
<u>Refer to video example 2.</u>

The diagram shows the plan for a triangular piece of mobile. Where should the sculptor attach the support so that the triangle is balanced?



2 Problem-Solving Application

The diagram shows the plan for a triangular piece of a mobile. Where should the sculptor attach the support so that the triangle is balanced?



1 Understand the Problem

The **answer** will be the coordinates of the centroid of $\triangle PQR$. The **important information** is the location of the vertices, $P(3, 0)$, $Q(0, 8)$, and $R(6, 4)$.

2 Make a Plan

The centroid of the triangle is the point of intersection of the three medians. So write the equations for two medians and find their point of intersection.

3 Solve

Let M be the midpoint of $\overline{QR}$ and N be the midpoint of $\overline{QP}$.

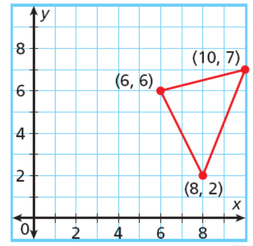
$$M = \left(\frac{0+6}{2}, \frac{8+4}{2}\right) = (3, 6) \qquad N = \left(\frac{0+3}{2}, \frac{8+0}{2}\right) = (1.5, 4)$$

$\overline{PM}$ is vertical. Its equation is $x = 3$. $\overline{RN}$ is horizontal. Its equation is $y = 4$. The coordinates of the centroid are $S(3, 4)$.

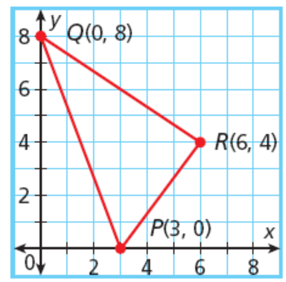
4 Look Back

Let L be the midpoint of $\overline{PR}$. The equation for $\overline{QL}$ is $y = -\frac{4}{3}x + 8$, which intersects $x = 3$ at $S(3, 4)$.

Example 2. A sculptor is shaping a triangular piece of iron that will balance on the point of a cone. At what coordinates will the triangular region balance?

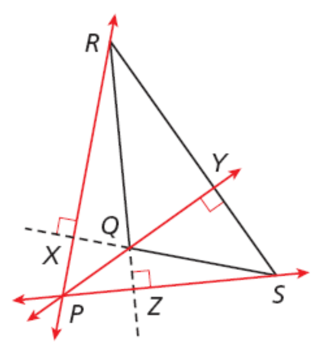


6. **Guided Practice.** Find the average of the *x*-coordinates and the average of the *y*-coordinates of the vertices of ΔPQR. Make a conjecture about the centroid of a triangle.



An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.
Every triangle has three altitudes. An altitude can be inside, outside, or on the triangle.

In ΔQRS, altitude $\overline{QY}$ is inside the triangle, but $\overline{RX}$ and $\overline{SZ}$ are not. Notice that the lines containing the altitudes are concurrent at P. This point of concurrency is the **orthocenter of the triangle**.

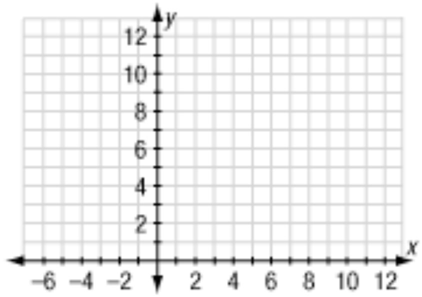


Helpful Hint

The height of a triangle is the length of an altitude.

<u>Refer to video example 3.</u>

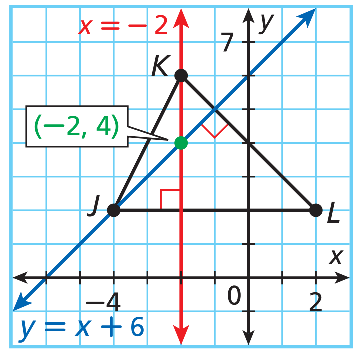
Find the orthocenter of $\triangle JKL$ with vertices $J(-6, 0)$, $K(12, 0)$, and $L(0,12)$.



3 Finding the Orthocenter

Find the orthocenter of $\triangle JKL$ with vertices $J(-4, 2)$, $K(-2, 6)$, and $L(2, 2)$.

Step 1 Graph the triangle.



Step 2 Find an equation of the line containing the altitude from K to $\overline{JL}$.

Since $\overleftrightarrow{JL}$ is horizontal, the altitude is vertical. The line containing it must pass through $K(-2, 6)$, so the equation of the line is $x = -2$.

Step 3 Find an equation of the line containing the altitude from J to $\overline{KL}$.

$$\text{slope of } \overleftrightarrow{KL} = \frac{2-6}{2-(-2)} = -1$$

The slope of a line perpendicular to $\overleftrightarrow{KL}$ is 1. This line must pass through $J(-4, 2)$.

$y - y_1 = m(x - x_1)$	*Point-slope form*
$y - 2 = 1[x - (-4)]$	*Substitute 2 for y_1, 1 for m, and −4 for x_1.*
$y - 2 = x + 4$	*Distribute 1.*
$y = x + 6$	*Add 2 to both sides.*

Step 4 Solve the system to find the coordinates of the orthocenter.

$$\begin{cases} x = -2 \\ y = x + 6 \end{cases}$$

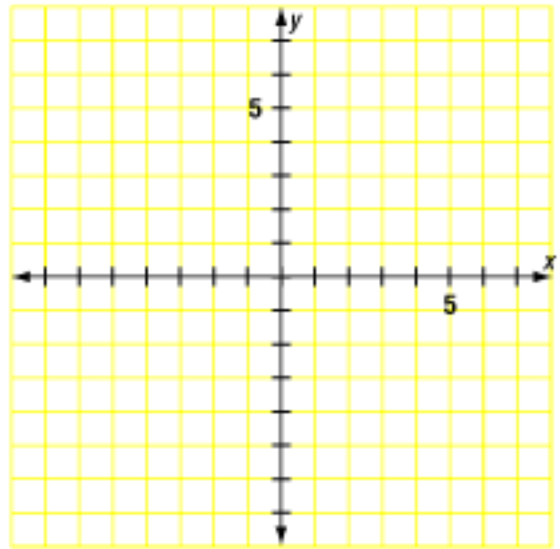
$y = -2 + 6 = 4$ *Substitute −2 for x.*

The coordinates of the orthocenter are $(-2, 4)$.

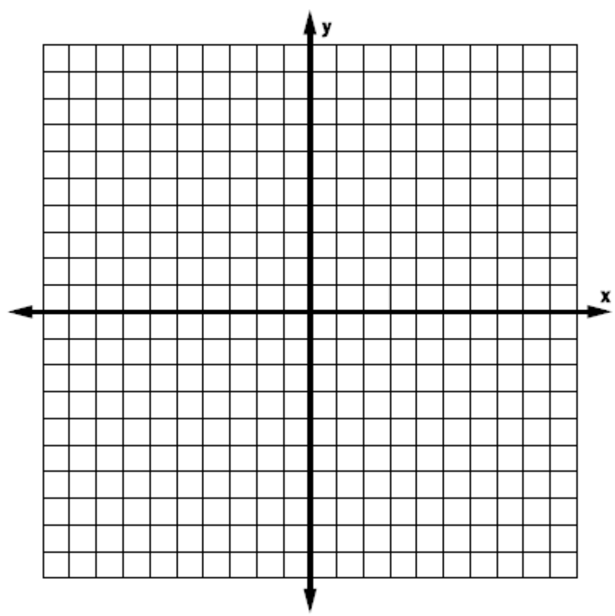
Q: What do you do when it rains?
A: Coincide!

On a clear disk you can seek forever.

<u>Example 3</u>. Find the orthocenter of ΔXYZ with vertices $X(3, -2)$, $Y(3, 6)$, and $Z(7, 1)$.



7. <u>Guided Practice</u>. Find the orthocenter of a triangle with vertices of P(-5, 8), Q(4, 5), & R(-2, 5)



5-3 Medians and altitudes of a triangle *(pp 330)* 13, 14, 16, 17, 19, 22, 29-37, 40.